

# The Mechanics of Moving Asteroids\*

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The fundamental problem for all asteroid mitigation concepts is how to best alter the trajectory of the hazardous body. This is a non-trivial issue and brings together many different aspects of dynamics, engineering, and small body science. This paper will focus on some select issues that pertain to this problem, and will attempt to define a coherent approach to a class of solutions to this problem. First, we will discuss what is currently known or suspected about the surface, interiors and spin states of small bodies, and the implications of these. Second, we will discuss the mechanics of altering an asteroid's trajectory and spin state, incorporating realistic numbers for thrust levels available. Finally, combining these previous topics, we will discuss some possible approaches to implementing these maneuvers. Throughout we will focus on the use of continuous thrust devices for effecting these changes to the small body state.

## Introduction

In this paper we address a simple question: what are the basic mechanics of moving an asteroid? This turns out, in fact, to be a complex issue involving non-trivial interactions between two 6DOF bodies, the spacecraft (or tug) and the asteroid itself. Simple analyses performed for moving point mass bodies do not hold up, in general, when applied to the realistic problem of pushing on a non-spherical, rotating asteroid. In this paper we explore different options for generating low-level, controlled propulsion of an asteroid over long time spans.

When confronting an asteroid spinning freely in space one is dealing with an uncommonly huge object. The largest Nimitz class aircraft carrier in the U.S. Navy fleet displaces about 88,000 metric tons of water ( $88 \times 10^6$  kg). A typical 200 meter diameter asteroid with density  $2 \text{ g/cm}^3$  weighs in at  $8.4 \times 10^9$  kg, or almost 100 times the mass of the worlds largest aircraft carrier. Maneuvering an object of this size with a propulsion system which might be able to exert 10 newtons of force is a daunting challenge indeed!

In the typical case we are also dealing with a spinning object. Most asteroids larger than 200 meters spin at a rate below that which would cause an object on its equator to experience weightlessness. This translates to rotation rates below 12 revolutions/day (2 hr/rev) with most having rotation periods of 4 hours or greater. While most asteroids rotate steadily about their maximum moment principal axis, some pose a special challenge by tumbling randomly.

Using the assumption that one intends to deflect an incoming NEA 10 years or more before impact, the optimum  $\Delta V$  aligns with the velocity vector, either posigrade (to cause it to rendezvous too late) or ret-

rograde (to cause it to rendezvous too early). The challenge then becomes one of applying the available force on the asteroid for an adequate period of time to change the asteroid's velocity by the desired amount to miss the Earth. For a typical time-to-impact of 10 years, a properly oriented  $\Delta V$  of one or two cm/sec would be adequate to cause the asteroid to miss its rendezvous with the Earth. Putting aside, for the moment, the major issue of attachment of the vehicle or thrusting mechanism to the asteroid, the specific design challenge is to accelerate the asteroid along its velocity vector while it is rotating about an axis randomly oriented in space.

In this paper we review several options for the solution of this challenge, describing in more detail the decision process that the B612 Foundation went through in developing their implementation plan, partially reported in.<sup>10</sup> One issue that we do not address is the effect of uncertainty in the asteroid's orbit on the optimal direction in which to move it. This is a crucial consideration and thus deserves a full discussion on its own. We refer the reader to the paper by Chesly and Spahr<sup>3</sup> for an introduction to some of the issues related to this concern.

In summary, the procedure that we suggest for deflecting an asteroid allows us to continually tug the asteroid in the optimal direction, along its velocity vector, while simultaneously causing the asteroid rotation axis to precess in order to allow for this optimal thrusting to continue. This approach can be generically applied to any size asteroid. Furthermore, this can be achieved by attaching the spacecraft tug to the asteroid with a single cable attached at the asteroid rotation pole. This reduces the interaction between the tug and asteroid to a single universal joint anchored (or otherwise attached) to the asteroid rotation pole. The background, derivation, and assumptions made in developing this concept are outlined below in detail.

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## Survey of asteroid properties

We first review the basic mechanical and surface properties of asteroids. A comprehensive review of the asteroid environment is found in the book *Asteroids III*,<sup>2</sup> and much of the information we quote below is taken from this reference.

### Asteroid sizes and shapes

Potentially hazardous Near Earth asteroids, as a total population, span a large range of sizes, from bodies that are tens of meters across to asteroids the size of Eros, over 35 km in length. Currently, it is believed that the largest asteroids have been detected in the Near Earth Object (NEO) population down to kilometer-sized objects, leaving the smaller bodies of 100 meters to 1 km size to populate the immediate hazard. Still, among this class of smaller asteroids one finds a great diversity of shapes and morphologies.

In this regime, the best data on asteroid shapes and sizes comes from the field of radar astronomy. There have been a number of published shape models for asteroids in the size range of a few km or smaller.<sup>7</sup> A partial list of asteroids for which shape models exist, or soon will exist, includes 4769 Castalia, 1620 Geographos, 4179 Toutatis, 1998 ML14, 25143 Itokawa, 6489 Golevka, 2063 Bacchus, and 1998 KY26, while 66391 (1999 KW4) and 2000 DP107 are binary asteroids. A brief visual survey shows that they exhibit a range of shapes, from a near-spherical object (1998 ML14), to a bifurcated object (Castalia), to a extremely distorted body (Geographos), to a more simple ellipsoidal shape (Itokawa). Any mission to an asteroid must be prepared to deal with this range of variability, and must be robust to it. This being said, in this paper we do not consider binary asteroids in any special detail. We do note that the operations we discuss could all be carried out on the larger of the two bodies in a binary asteroid pair, however. While we have not analyzed it, we believe that the smaller body in a binary pair would be dragged along with the larger.

To continue our discussion we must define a number of properties for these small bodies. First, we define the mass distribution of the asteroid as a set  $\mathcal{B}$  which consists of all points contained within the asteroid. To every vector  $\mathbf{r}$  contained in  $\mathcal{B}$  we can assign a density,  $\rho(\mathbf{r})$ , thus realizing that a natural body will not, in general, have a constant density. We do note, however, that the asteroid Eros was determined to have a near-constant density distribution.<sup>6</sup> With this definition we can define the total volume and total mass of the body:

$$V = \int_{\mathcal{B}} d\mathbf{r}^3 \quad (1)$$

$$M = \int_{\mathcal{B}} \rho(\mathbf{r}) d\mathbf{r}^3 \quad (2)$$

Two useful numbers that help characterize an asteroid

are its mean radius and mean density, defined as:

$$r_o = \left( \frac{3V}{4\pi} \right)^{1/3} \quad (3)$$

$$\rho = \frac{M}{V} \quad (4)$$

A related number of interest for orbital operations is the total gravitational parameter of an asteroid,  $\mu$ , computed as the gravitational constant  $G = 6.672 \times 10^{-8} \text{ g/s}^2/\text{cm}^3$  times the mass, or  $\mu = GM$ . While the gravity field of an asteroid in general deviates considerably from an ideal point mass, it is still useful to use a point mass potential to determine the basic orbital mechanics properties of motion about an asteroid.

For the work we are considering here, another important concept is the inertia moments of the asteroid, as they determine how its rotational motion evolves. Given the concepts mentioned above, the inertia matrix of an asteroid is defined as:

$$\mathcal{I} = \int_{\mathcal{B}} [\mathbf{r}^T \mathbf{r} \mathbf{I} - \mathbf{r} \mathbf{r}^T] \rho(\mathbf{r}) d\mathbf{r}^3 \quad (5)$$

The inertia is a symmetric 3x3 matrix in general, however a set of axes can always be found such that the inertia is a diagonal matrix. These axes are defined as the principal axes of the body, and the related moments of inertia are designated as  $I_1 \leq I_2 \leq I_3$ . Even though the principal axes and moments of inertia of an asteroid always exist, they are not always easy to find. An exception is for an asteroid uniformly rotating about its maximum moment of inertia, as the rotation pole will lie through the corresponding principle axis. For any other situation, the estimation of principal axes is much more difficult.

### Asteroid rotation states

The second crucial property of asteroids is that they all rotate. Their range of rotation periods span many orders of magnitude, with the fastest rotators making one revolution every few minutes and the slowest rotators taking days or weeks to make one full rotation. Furthermore, while most asteroids are in a principal axis rotation state about their largest moment of inertia ( $I_3$ ), a non-negligible fraction of them are in a complex rotation state, tumbling in space. Such tumbling motion seems to be most frequently associated with slow rotators such as Toutatis,<sup>4</sup> although there have also been small fast rotators detected which are tumbling. Across the NEO population the average rotation period is approximately 5 hours, and the vast majority of asteroids are in uniform rotation about their largest moment of inertia. The one definitive statement one can make about rotation rates is that the fast rotators (those with rotation periods less than  $\sim 2$  hours) are all small bodies, most likely monolithic shards of larger asteroids.

The rotational dynamics of an asteroid can be defined with two, related vectors. First is the angular velocity of the body, designated as  $\omega$ . The vector defines the instantaneous rotation axis and rotation rate of the body. Second is the angular momentum of the body, computed as the product of the inertia matrix and the angular velocity vector,

$$\mathbf{H} = \mathcal{I}\omega \quad (6)$$

We note that the angular momentum vector of a rotating asteroid is constant over the time scales of interest to us, however the same is not necessarily true of the angular velocity vector. If the asteroid is spinning about one of its principle axes (assume  $I_3$ ) then the angular momentum and angular velocity are simply proportional to each other, and the rotation rate is a constant. If the rotational velocity is not aligned with a principal axis, then it varies as a function of time in general, causing the asteroid to tumble in inertial space instead of having a simple, repeating rotation. In this paper we will focus on the case of a uniformly rotating body.

If the body is spinning about its maximum moment of inertia,  $I_3$ , and has an equatorial cross-section of  $2\alpha$  by  $2\beta$ , its moment of inertia can be estimated as:

$$I_3 \sim \frac{M}{5}(\alpha^2 + \beta^2) \quad (7)$$

and we note that this moment of inertia will always be greater than the moment of inertia associated with a sphere with mean radius  $r_o$  and mass  $M$ , or:

$$I_3 \geq 2Mr_o^2/5 \quad (8)$$

which is a useful lower bound.

### Asteroid surfaces and interiors

Asteroid surfaces are not well understood. When the NEAR spacecraft descended to the surface of asteroid Eros, it found that at the smallest scale, the surface was essentially free of craters.<sup>11</sup> This was not expected, and is in direct contrast to the surface of the moon, which has craters down to the smallest scales. For the asteroid Eros this directly implies a surface of loose material over the body, termed regolith, as well as a process that transports this material. Both of these observations are of importance for a mitigation mission to an asteroid, as discussed later. At the other end of the spectrum, asteroids such as 1998 KY26,<sup>8</sup> with its extreme rotation rate, are spinning so fast that they clearly cannot retain any surface material. In between these extremes are asteroids such as Golevka,<sup>5</sup> which has regions which are steep enough so that the presence of loose material is unlikely, yet may very well have other regions blanketed in regolith (see Fig. 1). Any pre-planned mitigation mission which interacts with an asteroid surface must be able to handle, in principle, all of these extremes.

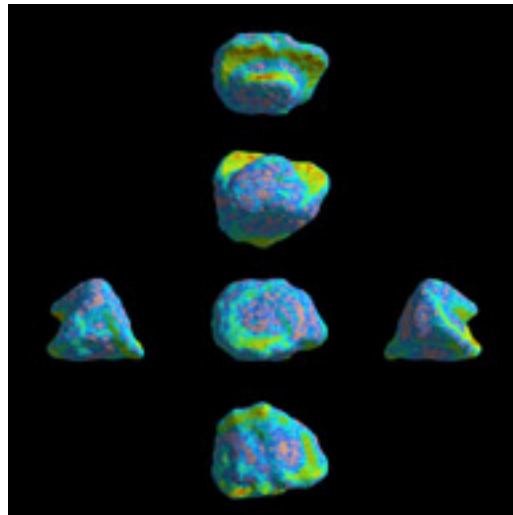


Fig. 1 Surface slopes of the asteroid Golevka.

Furthermore, there are currently no measurements of the sub-surface properties of asteroids. The best we can do is to infer, from the population of fast rotating monoliths, that within larger asteroids there exists a sub-structure of boulders of size a few hundred meters and less. It is important to note that there are no large asteroids with extremely rapid rotation rate, which implies that all larger bodies are probably comprised of fractured rock, which is in turn consistent with numerical simulations of impacts between asteroids.<sup>1</sup> The probable existence of a surface regolith lying over such a sub-stratus of boulders adds to what will be a challenging environment to affix a space structure to.

One additional possible implication of surface regolith and an associated transport phenomenon is that surface operations will cause clouds of regolith to be raised during surface interactions.<sup>9</sup> If the transport mechanism is due to charged particles, it is also possible that the spacecraft themselves may attract particles, which could be quite problematic. These issues of asteroid surface and sub-surface must be addressed by scientific missions to a representative sample of asteroids prior to a mitigation mission that directly interacts with the body surface. An incomplete or wrong understanding of this environment could be catastrophic for a large scale mission to the surface of an asteroid.

### Definition of a “Model” asteroid

Given the above discussions, we note that the likely range of asteroid parameters could be quite large. A complete study of the mitigation hazard would have to incorporate this full range of possibility. To focus our current work, however, we will define a specific model asteroid and use this for our example computations. As our model, we will choose a body with a mean radius  $r_o$  of 100 m, a density of  $\rho = 2 \text{ g/cm}^3$ , and a rotation period of 5 hours. From this, we can derive a number of other constants, including the total

mass  $M$ , the moment of inertial (computed assuming a spherical body)  $I_3$ , the angular velocity,  $\omega_o$ , and the angular momentum magnitude  $H_o$ :

$$\begin{aligned} r_o &= 100 \text{ meters} \\ \rho &= 2000 \text{ kg/m}^3 \\ P &= 5 \text{ hr} \\ \omega_o &= 3.5 \times 10^{-4} \text{ rad/s} \\ M &= 8.4 \times 10^9 \text{ kg} \\ I_3 &= 3.35 \times 10^{13} \text{ kg m}^2 \\ H_o &= 1.17 \times 10^{10} \text{ kg m}^2/\text{s} \end{aligned}$$

## Mechanics of changing an asteroid trajectory and rotation state

In the following sections we discuss the detailed mechanics involved in moving an asteroid. We see that we must concern ourselves with both the translational and rotational aspects of the problem. We start with a discussion of the optimal approach to effect a small deflection in an asteroid's trajectory, and end with a discussion of how to achieve such a deflection by controlling the orientation of the asteroid's spin axis while simultaneously pushing or pulling it along its path.

Before we get into a detailed discussion, we first give a very brief statement of the relevant equations for the translation and rotation of an asteroid subject to thrusting. First, the translational motion of the asteroid will be governed by the equations of motion:

$$\ddot{\mathbf{R}} = -\frac{\mu_S}{R^3}\mathbf{R} + \frac{1}{M}\mathbf{T} \quad (9)$$

where  $\mu_S$  is the sun's gravitational parameter,  $\mathbf{R}$  is the position vector of the asteroid in an inertial frame,  $M$  is the asteroid mass, and  $\mathbf{T}$  is the thrust vector applied to the asteroid. An important consideration to recall is the specific energy of the asteroid's orbit:

$$E = \frac{1}{2}\mathbf{V} \cdot \mathbf{V} - \frac{\mu_S}{R} \quad (10)$$

$$= -\frac{\mu_S}{2a} \quad (11)$$

where  $\mathbf{V} = \dot{\mathbf{R}}$  is the velocity of the asteroid and  $a$  is the semi-major axis of the asteroid's orbit. Under the presence of thrusting, the energy of the orbit will no longer be constant, but will have a time rate of change that is computed as:

$$\dot{E} = \mathbf{V} \cdot \mathbf{T}/M \quad (12)$$

In general we will assume that the asteroid has a circular orbit, meaning that its radius and speed are constant and equal to  $a$  and  $\sqrt{\mu_S/a}$ , respectively. Associated with this, the asteroid will have a constant angular rate as it moves about the sun called the "mean motion"  $n$  and equal to:

$$n = \sqrt{\frac{\mu_S}{a^3}} \quad (13)$$

It is important to note that the velocity vector will also rotate at this rate as the asteroid orbits about the sun. In the ensuing discussion we only consider circular orbits for convenience, all of our computations and analyses could be generalized to eccentric orbits as well.

For the rotational motion of the asteroid, the relevant equation involves the time rate of change of the angular momentum vector:

$$\dot{\hat{\mathbf{H}}} = \mathbf{M} \quad (14)$$

where  $\mathbf{M}$  is the moment applied to the asteroid. In our applications we will assume that the moment is due to a thrust applied at the surface of the asteroid, located at a position vector  $\mathbf{r}$ , thus the moment will equal:

$$\mathbf{M} = \mathbf{r} \times \mathbf{T} \quad (15)$$

We can decompose the angular momentum vector into its magnitude and direction:  $\hat{\mathbf{H}} = H\hat{\mathbf{H}}$ . Furthermore, if we assume principal axis rotation (which we do in this paper), the angular momentum magnitude is simply  $H = I_3\omega$  and the time rate of change of this is  $I_3\dot{\omega}$ . Thus, when considering the time rate of change of the angular momentum vector we must account for variations in both the magnitude and direction. Thus the general rotational equation of motion we consider is:

$$I_3\dot{\omega}\hat{\mathbf{H}} + H\dot{\hat{\mathbf{H}}} = \mathbf{r} \times \mathbf{T} \quad (16)$$

Using these equations, we can now discuss the dynamics of deflecting and precessing an asteroid.

### Deflecting an asteroid

Should a hazardous asteroid ever be detected, chances are that we will have a relatively long lead time until impact occurs, although this is not guaranteed.<sup>3</sup> Assuming this situation, what is really needed to cause a potentially hazardous asteroid to miss the Earth is a small shift in its orbital mean motion, as this will cause it to slow or speed itself in its orbit and miss the Earth. If the time to impact becomes small then it may become optimal to shift the trajectory in a direction that does not necessarily change its mean motion. Although we do not assume that situation here, all the following material could be changed to accommodate this situation as well.

Given a change in the mean motion of the asteroid, the total shift in the asteroid's position along its orbit can then be approximated as the change in mean motion, times the time till close Earth approach, times the asteroid semi-major axis. Thus, we find the shift in the asteroid from its nominal (assumed impacting) trajectory to be  $\Delta s = \Delta n a t_{imp}$ . This is an approximate result, however, and will be corrected below. But we see immediately that we must also specify the necessary shift in the trajectory.

The amount by which the asteroid path needs to be shifted is, at most, equal to the radius of the Earth times a focusing factor which depends on the approach hyperbolic speed of the asteroid. This focusing factor is:

$$f_\infty = \sqrt{1 + \frac{2\mu_E}{R_E V_\infty^2}} \quad (17)$$

where  $\mu_E$  is the Earth's gravitational parameter ( $\sim 4 \times 10^5$ ),  $R_E$  is the Earth's radius ( $\sim 6400$  km), and  $V_\infty$  is the asteroid's approach speed to the Earth. The total shift necessary is then computed as:

$$\Delta s = R_E f_\infty \quad (18)$$

For an approach speed of 10 km/s,  $f_\infty \sim 1.5$ , leading to a necessary shift of almost 10,000 km. If the asteroid approach speed is very slow, say on the order of 4 km/s, then this factor increases to 3, leading to a necessary shift of almost 20,000 km. Clearly, such a slow approach speed would be a serious problem for mitigation, however any asteroid on such a low energy approach would most likely be of smaller size. For our example we use a target value of 10,000 km, realizing that all of our results will scale non-linearly with the  $V_\infty$  of the approaching asteroid.

Now, let us consider, more accurately, the effect of thrusting on the shift in the asteroid's path. The angular rate of the asteroid orbit (assuming a circular orbit for simplicity) equals its mean motion, or:

$$\dot{\theta} = n \quad (19)$$

Thus the angular acceleration equals

$$\ddot{\theta} = -\frac{3n}{2a}\dot{a} \quad (20)$$

where  $\dot{a}$  can be computed from the energy equation to be

$$\dot{a} = \frac{2a^2}{\mu_S} \dot{E} \quad (21)$$

If we assume that we thrust along or against the velocity of the asteroid we have an acceleration rate  $A = T/M$ , and the time rate of change of the energy is

$$\dot{E} = \pm VA \quad (22)$$

respectively, where  $V$  is the speed (assumed local circular again). Resulting from this, we find that:

$$\ddot{\theta} = \mp 3 \frac{A}{a} \quad (23)$$

which is a familiar, if not counter-intuitive, result. Namely, a constant increase in the energy of the vehicle will cause the asteroid to fall behind its nominal

trajectory, and vice-versa. Assuming a constant acceleration, the general solution for  $\theta$  and  $\dot{\theta}$  is:

$$\theta = \theta_o + \dot{\theta}_o t \mp \frac{3A}{2a} t^2 \quad (24)$$

$$\dot{\theta} = \dot{\theta}_o \mp 3 \frac{A}{a} t \quad (25)$$

Given these results, the accumulated change in path length is then computed as  $\Delta s = a\theta$  and the accumulated change in speed is  $\Delta V = a\dot{\theta}$ .

Now, let us calculate the total deflection of the asteroid. First, assume that a constant thrust is applied over a time  $t_a$ . Then at the end of this period the total shift in position and speed will be

$$\Delta s = \mp \frac{3}{2} A t_a^2 \quad (26)$$

$$\Delta V = \mp 3 A t_a \quad (27)$$

If this period of acceleration is followed by a long coast time,  $t_c$ , the accumulated change in path length is:

$$\Delta s = \mp \frac{3}{2} A t_a (t_a + 2t_c) \quad (28)$$

To achieve a 10,000 km shift in 10 years we consider two scenarios. First we assume a constant acceleration for the entire time,  $t_a = 10$  years,  $t_c = 0$ . The necessary acceleration is then  $6.75 \times 10^{-11}$  m/s<sup>2</sup>. Although extremely small, this is still orders of magnitude larger than the solar radiation pressure acceleration acting on our example body. In terms of applied thrust to the asteroid, we multiply the acceleration by the mass of the asteroid to find a thrust  $T \sim 0.6$  N to move a 100 m radius asteroid by the requisite amount in 10 years. Although the engines are required to operate over 10 years, the level of thrusting required is clearly within our capability.

An alternate strategy would be to thrust for a shorter time period with a stronger engine, followed by a coast period. Let us set the acceleration time to be one year and the coast time to be 9 years. Then the required acceleration to shift the asteroid by 10,000 km would be  $3.5 \times 10^{-10}$  m/s<sup>2</sup>, or a thrust on the order of 3 N.

This is a "best case" analysis in that we assume thrusting along the asteroid velocity vector over the entire time span. This is a challenging thing to do, however, given that the asteroid rotates once each asteroid day and that its velocity vector rotates through 360 degrees every asteroid year. If our thrust vector is directly fixed on the asteroid surface through the asteroid center of mass, it will be acceptably aligned with the velocity vector a vanishingly small portion of the time due to asteroid rotation and asteroid motion around the Sun. Therefore we have to look at options to control the asteroid.

## Direct control of an asteroid

One possible way in which to implement a capability for continuously changing the thrust direction is to turn the entire asteroid into a “vehicle” which the thrust system re-orient as it thrusts. To do this would require that the asteroid first be “de-spun” and the thrusting engine be subsequently situated so that it thrusts through the asteroid center of mass and controls the asteroid orientation by properly gimbaling its thrusting engines. This is entirely feasible, as an asteroid could be de-spun by placing a thruster at the equator and applying a torque opposite to the rotation pole (see Fig. 2). For this maneuver, the time rate of change of the angular momentum magnitude due to a constant thrust located at a height  $h$  above the equator is:

$$I_3\dot{\omega} = -(r_o + h)T \quad (29)$$

Assuming a constant thrust, the angular velocity of the asteroid as a function of time will be:

$$\omega - \omega_o = -\frac{(r_o + h)T}{I_3}t \quad (30)$$

Thrusting until the body is no longer spinning yields the de-spin time,  $t_D$ :

$$t_D = \frac{I_3\omega_o}{(r_o + h)T} \quad (31)$$

For a 5 N thruster placed at a height  $h = r_o$  above our model asteroid (100 m), the time to despin is 4.5 months.

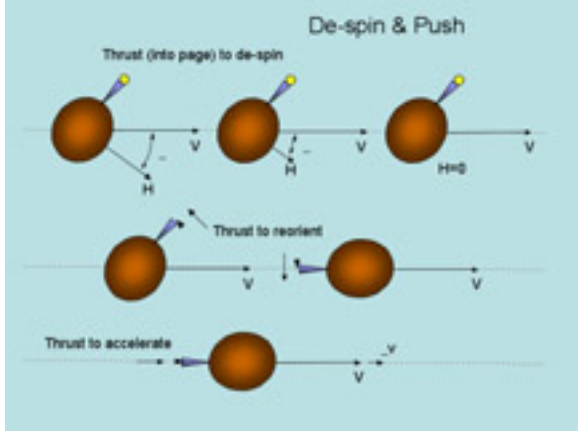


Fig. 2 De-spin and push.

Then, to re-orient the vehicle as it thrusts would require an ability to turn the asteroid at a rate of approximately 1 degree per day (assuming a circular orbit at  $\sim 1$ AU). This can be achieved by inducing the asteroid to spin at the appropriate rate about a pole normal to its orbit plane, and adding in additional control accelerations as needed. The level of thrusting authority to implement this is relatively small. The asteroid attitude will need to be controlled, however,

since if rotation is induced about a non-principal axis the asteroid will slowly tumble unless corrected by the thruster system. While this would be a challenging control problem, it is not the most severe problem that this approach faces.

If the body in question is not a monolith, then as it is despun its internal stress distributions will be significantly changed.<sup>12</sup> We must recall that the spinning body has naturally achieved some equilibrium between its self-gravitational attraction, its centrifugal accelerations, and its internal stress distribution. A large change in its rotation rate will destroy this equilibrium and create a body that is out of equilibrium. As the asteroid is likely to be composed of a collection of smaller boulders, this body will most likely be subject to large shifts in the relative positions between its components as its spin rate decreases. Any one of these shifts would be likely to destroy the attached propulsion system, or at the least create a hazardous environment as the components of the body rearrange themselves. Following such a shift, it is likely that the asteroid components may impact and even be in close orbits about the body for a period of a few days. In addition to these hazards, once the body has settled into a new equilibrium its moments of inertia and center of mass location must all be re-determined, a process that may be more difficult for a non-rotating body.

If the body is clearly a monolith (i.e., is rotating rapidly) these concerns may not be as much of an issue. Then, however, it will require a longer time to de-spin the body. For our example of a 100 meter radius body, a 5 N thruster, and an altitude of  $h = 100m$ , an initial spin period of 60 minutes would take over 1.5 years to despin. Also, it may be more difficult to anchor a tall structure at the equator, as there will be a strong centrifugal acceleration.

## Precession of an asteroid spin state

An alternative to despinning the asteroid is instead to precess its spin axis in inertial space, much like a spinning spacecraft’s orientation angle can be modified. The implementation of such maneuvers for an asteroid can be envisioned as follows. The propulsion unit is placed at one of the poles of the asteroid, and has a degree of freedom allowing it to thrust perpendicular to the rotation pole in any direction. Specifically, as the asteroid rotates it is able to continually thrust in the same direction in inertial space. Considering this, the resultant moment produced by the thruster can be computed as:

$$\mathbf{M} = r_o T \hat{\mathbf{H}} \times \hat{\mathbf{H}}_{\perp 1} \quad (32)$$

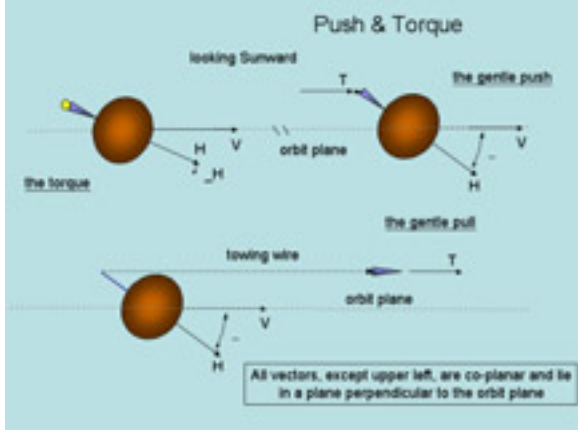
where  $\hat{\mathbf{H}}_{\perp 1}$  is a unit vector perpendicular to the asteroid angular momentum vector. Under this moment, the magnitude of the angular momentum will not change, so the equation of motion becomes:

$$H\dot{\sigma}_2 \hat{\mathbf{H}}_{\perp 2} = r_o T \hat{\mathbf{H}}_{\perp 2} \quad (33)$$

where  $\hat{\mathbf{H}}_{\perp 2}$  is mutually perpendicular to the other two unit vectors and will be fixed in space as the other unit vectors rotate under the moment, and where  $\dot{\sigma}_2$  is the angular rate at which the asteroid angular momentum vector rotates (see Fig. 3). Thus, the precession rate and rotation angle of the asteroid angular momentum (and spin pole) will be:

$$\dot{\sigma}_2 = \frac{r_o T}{H_o} \quad (34)$$

$$\sigma_2 = \frac{r_o T}{H_o} t \quad (35)$$



**Fig. 3 Precess and push or pull.**

Using this approach, the time it takes to despin an asteroid is equivalent to the time it takes to precess the spin pole by 1 radian (57 degrees). The main advantage of this approach is that the total spin rate of the body is not changed, nominally, meaning that the asteroid will not be disturbed from its natural equilibrium state.

The speed with which the asteroid can be precessed is not as fast as the speed with which a non-spinning asteroid can be reoriented, however. To precess our model asteroid (100 meter radius, 5 hour period, altitude = 0) at a 1 degree per day rate will require 23 N of continuous thrust. However, it is clear that it is possible to re-orient the asteroid rotation pole over reasonable periods of time.

### Simultaneously pushing and precessing an asteroid

Now we will combine our approaches and search for a condition where it may be possible to tug the asteroid in the optimal direction (along the velocity vector) while maintaining a fixed orientation of the asteroid rotation pole relative to the velocity vector. We can do this if we induce the appropriate precession rate of the asteroid rotation pole as we thrust along the velocity vector.

First, we assume that we are able to precess the asteroid rotation pole so that it lies, at a given epoch, in whatever orientation relative to the asteroid orbit that is needed. We assume that the propulsive device

is situated at the asteroid pole, and that it is gimbed such that it can direct its thrust parallel to the asteroid velocity vector as the asteroid rotates. Thus, it continually adds or subtracts energy from the asteroid orbit, as desired. To analyze this system, we re-write the equations of rotational motion in a frame rotating with the asteroid orbit velocity vector. Choosing the orbit normal direction to be the  $\hat{\mathbf{z}}$  axis, this rotational velocity vector equals  $n\hat{\mathbf{z}}$ . Choose the  $\hat{\mathbf{x}}$  axis of this rotating frame along the asteroid velocity vector, and the  $\hat{\mathbf{y}}$  axis from the usual cross-product. Then, in this frame the thrust vector will, ideally, equal  $\mathbf{T} = T\hat{\mathbf{x}}$ , the position vector of the propulsive unit will equal  $r_o\hat{\mathbf{H}}$ , and the rotational equations of motion become:

$$\left(\dot{\mathbf{H}}\right)_r + n\hat{\mathbf{z}} \times \mathbf{H} = r_o T \hat{\mathbf{H}} \times \hat{\mathbf{x}} \quad (36)$$

The sub-script  $r$  denotes that the time derivative takes place in the rotating frame. To be able to simultaneously tug and precess the asteroid we wish to set  $\left(\dot{\mathbf{H}}\right)_r \equiv 0$ , as then this relative orientation will be maintained. This implies that  $\hat{\mathbf{z}} \times \mathbf{H}$  is parallel to  $\hat{\mathbf{H}} \times \hat{\mathbf{x}}$ , a situation that can only occur if  $\hat{\mathbf{H}}$  lies in the  $\hat{\mathbf{x}} - \hat{\mathbf{z}}$  plane. Let us assume so, and furthermore stipulate that the  $\hat{\mathbf{H}}$  vector makes an angle  $\sigma$  with the  $\hat{\mathbf{x}}$  axis. Then, we find that:

$$nH \cos \sigma \hat{\mathbf{y}} = r_o T \sin \sigma \hat{\mathbf{y}} \quad (37)$$

$$\tan \sigma = \frac{nH}{r_o T} \quad (38)$$

We note that for any level of thrust  $T$  there will always be a unique angle  $\sigma$  that will maintain our thrusting situation. Figure 3 shows a representation of this approach. For our example asteroid with a semi-major axis of 1 AU and a thrust of 10 N, this angle is 67 degrees, while for a 1 N thrust it equals 87.5 degrees. For the de-spun asteroid case considered earlier, we see that  $\sigma = 0$ .

Thus, across the spectrum of applied thrust levels, we see that we are always able to thrust along the velocity vector and precess the asteroid rotation pole along with us to maintain the appropriate geometry of the situation. This is a significant point, as it is now possible for us to simultaneously tug the asteroid and maintain a favorable orientation simultaneously. Furthermore, once the propulsion device is fixed to the pole it never needs to be moved, as the initial precession maneuver and the simultaneous period of precession and tugging can all occur relative to the one contact point. Thus, we have identified this sequence of maneuvers as, perhaps, the best manner in which to deflect the asteroid trajectory using a continuous thrust device.

### Implementation challenges

The above scenario has a number of obvious technical challenges, even beyond generating thrust for

sustained periods of time. However, by developing a methodology that only requires a single attachment point on the body at a well-defined location (i.e., the rotation axis), the overall complexity of our approach is reduced.

The fundamental implementation issue we must deal with is attaching the thrusting device to the asteroid. While asteroids may have regolith, the regolith is not expected to be particularly strong or able to sustain the stress necessary to hold an anchor, even at the small levels of thrust we are considering. Thus, if a direct anchoring scheme is proposed, it must involve driving a spike past the regolith and into “bedrock,” whatever that means on an asteroid. Due to the unknown surface and sub-surface properties of asteroids, it may be necessary to first survey a given asteroid to determine the feasibility of this approach.

Before we discuss anchoring, however, we should point out that there are two basic ways in which we can deliver thrust to our asteroid: pushing or pulling. Each of these approaches will have significantly different technological solutions. For pushing, we would affix a propulsive device along the rotation axis of the asteroid. The thrusting unit would need to have a gimble that would allow it to maintain a constant thrust in a given direction as the asteroid itself spins. The attachment device would have to maintain a compressive load and a bending moment, as the thrust will be an angle  $\sigma$  off of the rotation pole, in general. Both of these loads will be problematic. A compressive load of  $\sim 10$  N applied to the asteroid surface is much larger than any compression the body will likely have felt in the past (except if it is the core of a larger parent planetesimal). Thus there is a strong possibility for the material to undergo a long-term crushing or compaction response to this load, which would in turn cause misalignment of the propulsive structure from the rotation axis. While this can be mitigated by employing a large footpad, there will still be some level of crushing and settling that is likely to occur. There will also be a rotating bending moment applied across the spacecraft structure, and across the gimble axis. Thus to maintain its vertical integrity, the propulsive unit must also be braced against sway. Again, the devices used to implement this will need to sustain compressive or tensile loads that may never have been experienced by the surface material. An additional problem is the need for a gimbled engine, as it will be under a relatively heavy duty cycle when considered over the number of repetitions and the time span in the space environment. As the surface regolith will likely have relatively fine material, the prospect of this material contaminating the gimble bearings must also be considered.

If we instead consider pulling, we can get relief from some aspects of the technological problem. As seen in Fig. 3, it is now possible to not use a gimbled

engine, but instead to affix a universal joint to the surface, which is in turn attached to the tug by a cable. Again, the anchor must sustain loads on the order of 10 N, including periodic lateral loads that, over time, would be expected to loosen the anchor’s grip into the asteroid “bedrock”. The engine itself, however, will be much simpler and will not require a gimbled nozzle. Additionally, with this approach the spacecraft is able to easily make small adjustments to the thrust and torque it applies to the asteroid by changing its relative attitude to the asteroid.

Given the challenges of anchoring into the surface, an alternative approach has also been developed. Instead of anchoring a device into the asteroid, it is also possible to anchor it to the exterior of the asteroid by wrapping or enveloping the asteroid surface with sufficiently long “arms” that can distribute the surface load over a much greater area. There are many specific design approaches that can be considered in this vein, but they all would achieve gripping strength through some sort of grappling or wrapping around the asteroid surface. One particular concept, shown in Fig. 4, would have a central anchor for stabilizing the center of the attachment device, and then would deploy a series of arms long enough for them to wrap around the curvature of the asteroid. The more compressive contact points between the arms and the asteroid surface, the smaller the load each one must bear. Then, a universal joint at the center of the attachment device (and located at the asteroid rotation pole) is coupled to a cable that attaches to the spacecraft.

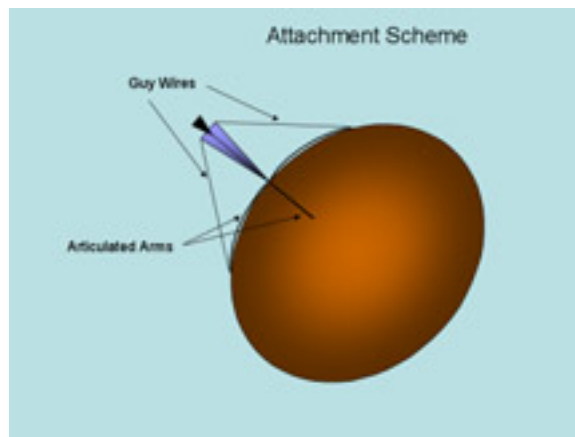


Fig. 4 Grapple.

## Conclusions

In this paper we have discussed the problem of deflecting an asteroid using a controlled thrust. We have clearly identified a sequence of maneuvers that addresses realistic deployment issues for attaching a tug to an asteroid and subsequently pulling on it to effect an asteroid deflection. Additionally, we have identified several specific technical challenges that must be addressed for such an application to be implemented.

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